ANALYZING THE FRACTAL BEHAVIOUR OF THE DISTRIBUTION POWER GRID IN THE CITY OF GRENOBLE- FRANCE

Yousra SIDQI
Univ. Grenoble Alpes – France
Yousra.sidqi@g2elab.grenoble-inp.fr

Nicolas RETIERE
Univ. Grenoble Alpes – France
Nicolas.retiere@g2elab.grenoble-inp.fr

Pierre FRANKHAUSER
Université de Franche-Comté – France
pierre.frankhauser@univ-fcomte.fr

Gilles VUIDEL
Université de Franche-Comté – France
gilles.vuidel@univ-fcomte.fr

ABSTRACT

We analyze the fractal properties of the distribution power grid of the city of Grenoble. We introduce the scaling behavior as a useful tool to compare the fractal behavior of multiple systems. We conduct a geographical concordance analysis between the power grid and both the road network and built-up areas of the city of Grenoble. We analyze the fractal behavior of electrical properties of the distribution network.

INTRODUCTION

In the wake of the rapid growth of distribution power grids, it became crucial to reconsider their structure in order to optimize their operations by making them more flexible and resilient. To be able to propose new architectures, which meet optimality criteria, e.g better service, lower costs and minimized power losses, it is important to understand the behavior of the existing power grid across scales. Fractal analysis is a powerful tool to understand network organizations at different scales. It allows, for power networks to verify the existence of hierarchical scaling laws in spatial distributions and physical properties. The paper will focus on the case of the city of Grenoble (Fig 1). We will use the data provided by data the local Distribution System Operator of the city : Gaz et électricité de Grenoble (GEG) . The data set at hands describes the medium voltage distribution system. The road network and built-up areas description are available in the Institut géographique national (IGN) open data. To verify to what extent such empirical structures follow fractal behavior, different analysis methods were developed [1]. Here, we will use the box counting method [1] to compute the fractal dimension based on the spatial properties of the grid. Since real world systems do not strictly follow a fractal law, we will use the curves of scaling behavior [2] to explore the deviations from a strict fractal law in both cases. In order to understand how spatial organization follows the same logic across scales of both power network and associated urban systems e.g the road network and built-up areas, we will conduct a concordance analysis [3] between the buildings, the road network and the power grid. This method allows a direct comparison across scales of different networks or of a network and a built-up space and helps analyze to what extend they are coherent and therefore enable optimizing the grid to get a better compatibility.

We will introduce finally the electrical measurements of fractal behavior.

Fig. 1 Grenoble's power network (blue line), road netword (red line) and built-up footprints (dotted pastel)

THEORETICAL BACKGROUND

Fractal analysis of power grids

Let us remind the definition of the fractal dimension. Fractals are self-similar. By zooming in or out from the structure, we always find the same shape. If we measure a property over a zone chosen arbitrarily of size l of the structure, the number M of elements follow the power law:

$$M(l) = l^{D_0} = M_0 \qquad (1)$$

The fractal dimension D_0 is defined as the exponent of this law. It characterizes the scaling property of the object over scales and as long as self-similarity holds, it is constant.

The box counting method is one of the most commonly used methods to measure the dimension of fractal objects[4;5]. Boxes of variable size ε are used to cover the objects. For each value ε, we count the number $N(\varepsilon)$ of non-empty boxes of side ε needed to cover the object of length L. For fractal structures the relation holds: $N(\varepsilon) = a\,\varepsilon^{-D_0}$ where $\varepsilon \to 0$. Taking the logarithm of the relation yields $Log\ N = \log a - D_0 \log \varepsilon$, a being the fractal measure of the object, which is often called the 'shape prefactor'[5;6]. Hence, by logarithmic transformation, we obtain a linear relationship where the slope value is the fractal dimension D_0[5]. When assuming a discrete series of values ε_i, we obtain:

$$D_0 = -\frac{log\ N(\varepsilon_{i+1}) - log\ N(\varepsilon_i)}{log\ \varepsilon_{i+1} - log\ \varepsilon_i} \quad (2)$$

In order to conduct this analysis on power grids, we define, according to equation (2), the box size ε as the impedance of power lines Z. Here we measure voltage drops ΔV hence the fractal dimension:

$$D_0 = -\frac{log\ \Delta V(Z_{i+1}) - log\ \Delta V(Z_i)}{log\ Z_{i+1} - log\ Z_i} \quad (3)$$

Scaling behavior

For a theoretical fractal, the fractal dimension D_0 is strictly the same all over the scales. However, we can suppose that real world objects do not follow strictly a fractal law. To explore if deviations from the fractal law exist, we use the curve of scaling behavior, which turned out to be efficient[1;2]. For this purpose, we compute for each value ε_i the slope values in the double logarithmic representation, according to equation (2), and obtain then a local scaling exponent$\alpha(\varepsilon_i)$. It is a constant value equal to D_0 for theoretical fractals, which does not necessarily hold for empirical structures. Hence, by representing the sequence of these slopes as a function of the size ε_i, we obtain the scaling behavior $\alpha(\varepsilon_i)$, which variations informs us about changes in the fractal behavior of the analyzed structure across scales. In order to establish a link between the analyzed objects, we compute the ratio of their scaling behavior [4]. E.g. linking built-up space (build) to a network (net) yields:

$$\beta_{build/net}(\alpha_i) = \frac{\alpha_{build}(\epsilon_i)}{\alpha_{net}(\epsilon_i)} = \frac{\log N_{build}(\epsilon_{i+1}) - \log N_{build}(\epsilon_i)}{\log N_{net}(\epsilon_{i+1}) - \log N_{net}(\epsilon_i)} \quad (4)$$

We call this type of representation the concordance analysis[7]. It shows how the objects follow the same logic of covering space at each scale. The β-values becomes equal to one if the logics are the same.

Generalized dimensions

One could not only measure the number of non-empty boxes but physical quantities such as the mass, the conductivity, voltage drops, etc. Hölder's theory allows to study the uniformity (or singularity) of these measurements in different points of the fractal object [8].

Multi-fractal objects can also be characterized using the generalized dimensions, defined as [9]:

$$D_q = \frac{1}{q-1}\frac{\log \mu(q,\varepsilon)}{\log(\varepsilon)} \quad (5)$$

$\mu(q,\varepsilon)$ is the qth moment defined[10]:

$$\mu(q,\varepsilon) = \sum_{i=1}^{N}\left(\frac{M_i}{M_0}\right)^q \quad (6)$$

M_i is the measure inside each box and M_0 is the total measure of the system at hands.

For $q = 0$, $D_0 = \frac{-\sum_{i=1}^{N} 1}{log(\varepsilon)}$, which is the box counting dimension as defined in equation (2).

CONCORDANCE ANALYSIS

In this section, we will compare the fractal behavior of the power grid of the city of Grenoble to the built-up areas then to the road network. We will use the concordance indicator as described in equation (4).

Power grid vs road network

The concordance indicator between buildings and the power grid is defined by:

$$\beta_{build/powergrid}(\alpha_i) = \frac{\alpha_{build}(\varepsilon_i)}{\alpha_{powergrid}(\varepsilon_i)} \quad (7)$$

In Fig. 2, at small scales (lower than 20m), the ratio between the dimension of the buildings and the power grid decreases quickly from 1.9 to 1.5. At these scales, the built-up area dimension is close to 2 (ε much lower than building size), whereas the power grid is almost linear, ε being smaller than ramification distances. At larger scales, the concordance indicator decreases a lot until reaching a value close to 1. It is due to the ramification of the electrical network to deliver electricity all over the covered area while minimizing its length. It is worth noting that lower values of the concordance parameter would mean that the network is overdeveloped compared to the built-up distribution. The planner's objective would be to maximize this indicator. In this case, historically, the power grid developed in Grenoble was mainly a 5.5 kV network but has expanded very quickly with the construction of new buildings in the 1960s to host the winter Olympics. This expansion added a 20 kV layer, making the network hypertrophied. The local DSO is in the process of switching from a mixed 20 kV and 5.5 kV network to an only 20 kV network by 2020, which is expected to fit more the expectations of planners from a concordance point of view.

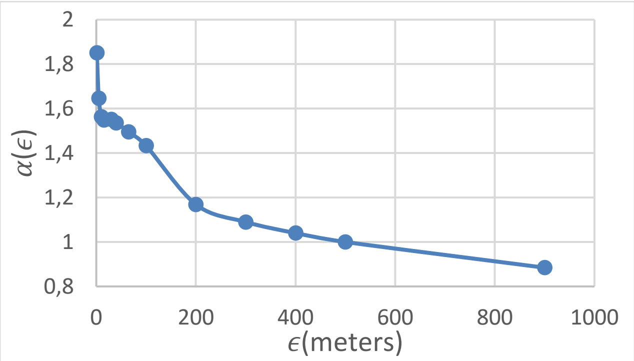

Fig. 2 Concordance analysis between the power grid and buildings

Power grid vs built-up areas

According to equation (4) we use the ratio defined by:

$$\beta_{road/power\ grid}(\alpha_i) = \frac{\alpha_{road}(\varepsilon_i)}{\alpha_{power\ grid}(\varepsilon_i)} \qquad (8)$$

At very local scales, this ratio is close to 1 (Fig.3) because both behaviors are linear. For ε between 0 and 20m, we notice a high rate of increase of β because the scaling behavior of the power grid remains almost linear (ε is smaller than its ramification distance) whereas the road network is already developed for enabling house accesses (Fig.1). For higher values of ε, both networks are rather in concordance, meaning that the power grid is rather hypertrophied.

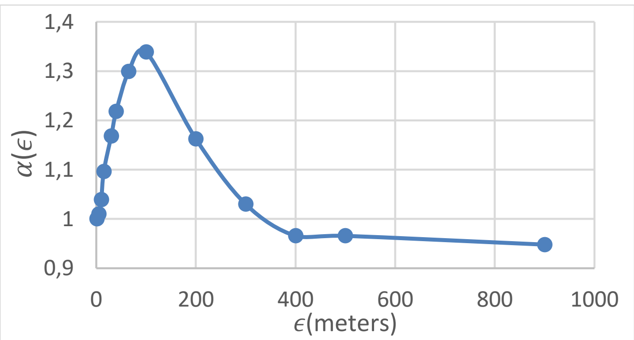

Fig. 3 Concordance analysis between the power grid and the road network

FRACTAL ANALYSIS OF VOLTAGE DROPS IN DISTRIBUTION POWER GRIDS

Using the theory at hand, there are several possible ways to quantify the "electrical singularity" of power grids: Active and reactive power consumed by every bus in the grid, voltage drops within the grid, energy losses. The main objective of this analysis is to assess the quality of power distribution.

In this research, the results of the AC load flow simulation are used to study the multi-fractality of the power grids. The line impedances are used as metric for graph traversal and voltages drops are assessed within the system.

Let's recall equation (6) to define the measure we are using to assess the electrical singularity. Here, we measure the distribution of voltage drops across the power grid.

$\mu_i(q,\varepsilon) = \sum_{i=1}^{N}\left(\frac{M_i}{M_0}\right)^q, \frac{M_i}{M_0}$ is the the sum of the voltage drops across branches in a cluster of radius ε centered around the node i. The measure is normalized using the total of voltage drops across branches of the network.

In Fig.4, we are assesing the distribution of the voltage drops for all the impedance values within the system. Typically, for each node, we are looking into how the voltage drops are distributed as we get further from it.

We notice that for small impedances, we are counting a small number of grid branches, which means the the voltage drop is rather small. As the impedances increase, the voltage drop increase in a exponential fashion, meaning that it might show some sort of fractal behavior across scales. For higher values, the voltage drop is equal the th whole system's meaning that we reached the borders of the system.

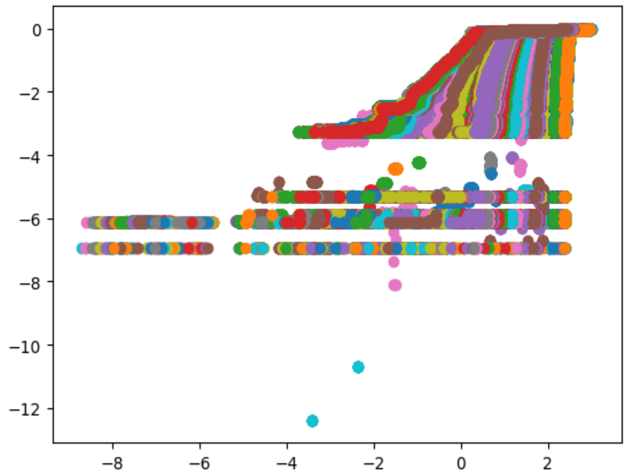

Fig. 4 Measures of $\mu_i(\varepsilon)$ in a double logarithmic projection (x axis is the metric which is the line impedance, y axis is the voltage drop for each node)

In Fig.5, we are calculating the q moments as shown in equation (6). For this purpose, we used an approached curve of Fig.4 using a linear regression. The local slopes of these results are then used to explore the electrical scaling behavior as introduced earlier.

Future work will utilize these results to conclude on the existence of a fractal behavior within electrical properties as seen geographically.

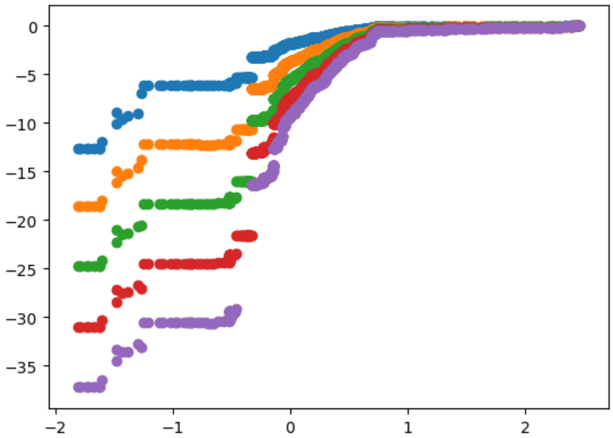

Fig. 5 Measures of $\mu_i(q,\varepsilon)$ in a double logarithmic projection (x axis is the metric which is the line impedance, y axis is q moment of the measures)

CONCLUSION

In this paper, we conducted a concordance analysis between the power grid, the road network and the buildings of the city of Grenoble, France. We used the box counting method to identify the geographical fractal behavior. We showed that the medium voltage power grid is overdeveloped as it is consistent with the road network and cover the built-up area in the same way. We introduced an adapted method for power grid to assess the distribution of voltage drops using different metrics. Future work will look further into the electrical scaling behavior of distribution power grids.

Acknowledgments

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